Application of Algorithmic Fuzzy Implications on Climatic Data.

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Abstract

Abstract

The main purpose of this paper is to find the appropriate fuzzy implication among the parametric and the three known (Kleen-Dienes, Lukasiewicz and Reichenbach) from the literature implications which will provide the smallest square error if adapted to our climatic data.

Objectives

1. As a first step a New Fuzzy Implication Generator was constructed via strong negations.
2. Then, an empiristic implication was constructed based on our data.
3. Finally, the pseudocode in the programming part of our paper used the New Fuzzy Implication Generator in order to approach the empiristic implication satisfactorily.

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Procedure of the first objective

Step 1

The new Fuzzy Implication Generator was constructed: via strong negations of Formula 1 which were constructed via conical sections ([Souliotis, G., Papadopoulos, B. 2019]).

$$N(x) = \sqrt{(\alpha^2 - 1)x^2 + 1} + \alpha \cdot x, \quad x \in [0,1], \quad \alpha \leq 0. \tag{1}$$

Procedure of the first objective

Step 2

via the

$$T_M(x, y) = \min\{x, y\} \quad (2)$$

In the future:

In future work we will use other t-norms in order to build new generators of parametric implications.

Procedure of the first objective

Step 3

via the corollary 2.5.31.
([Baczynski, Michał., Jayaram, Balasubramaniam. 2008]) which was Formula 3

$$I(x, y) = N(T(x, N(y))), \quad x, y \in [0, 1] \tag{3}$$

Procedure of the first objective

Step 4

via the combination of the above that created equation a which is algorithmic because the implication changed when parameter α changed. Parameter $\alpha \leq 0$

$$I(x,y) = \sqrt{(\alpha^2 - 1) . \left(\min\left(x, \sqrt{(\alpha^2 - 1)\, y^2 + 1} + \alpha \cdot y\right)\right)^2 + 1}$$
$$+\alpha \cdot \min\left(x, \sqrt{(\alpha^2 - 1)\, y^2 + 1} + \alpha \cdot y\right), y \in [0,1], \alpha \leq 0$$

(equation a)

Proof of the Equation a

1. $\left.\begin{array}{c} I(x,y) = N(T(x,N(y))) \\ N(x) = \sqrt{(\alpha^2-1)x^2+1}+\alpha\cdot x \end{array}\right\} \Rightarrow I(x,y) =$
$\sqrt{(\alpha^2-1)T^2(x,N(y))+1}+\alpha\cdot T(x,N(y))$
2. $\left.\begin{array}{c} I(x,y) = \sqrt{(\alpha^2-1)T^2(x,N(y))+1}+\alpha\cdot T(x,N(y)) \\ T_M(x,y) = \min\{x,y\} \end{array}\right\} \Rightarrow$
3. $I(x,y) = \sqrt{(\alpha^2-1).\left(\min\left(x,\sqrt{(\alpha^2-1)y^2+1}+\alpha\cdot y\right)\right)^2+1+}$
$\alpha\cdot\min\left(x,\sqrt{(\alpha^2-1)y^2+1}+\alpha\cdot y\right), y\in[0,1], \alpha\leq 0$

Proof of the Equation a

1. $\left.\begin{array}{c} I\left(\text{x,y}\right)=\text{N}\left(T\left(\text{x,N}\left(y\right)\right)\right) \\ N\left(x\right)=\sqrt{\left(\alpha^2-1\right)x^2+1}+\alpha\cdot x \end{array}\right\} \Rightarrow I\left(\text{x,y}\right)= \sqrt{\left(\alpha^2-1\right)T^2\left(\text{x,N}\left(y\right)\right)+1}+\alpha\cdot T\left(\text{x,N}\left(y\right)\right)$

2. $\left.\begin{array}{c} I\left(\text{x,y}\right)=\sqrt{\left(\alpha^2-1\right)T^2\left(\text{x,N}\left(y\right)\right)+1}+\alpha\cdot T\left(\text{x,N}\left(y\right)\right) \\ T_M\left(x,y\right)=\min\{x,y\} \end{array}\right\} \Rightarrow$

3. $I\left(x,y\right)=\sqrt{\left(\alpha^2-1\right).\left(\min\left(x,\sqrt{\left(\alpha^2-1\right)y^2+1}+\alpha\cdot y\right)\right)^2+1}+ \alpha\cdot\min\left(x,\sqrt{\left(\alpha^2-1\right)y^2+1}+\alpha\cdot y\right), y\in\left[0,1\right], \alpha\leq 0$

Proof of the Equation a

1. $$\left.\begin{array}{c} I\,(x,y) = N(T\,(x,N\,(y)) \\ N\,(x) = \sqrt{(\alpha^2-1)\,x^2+1} + \alpha \cdot x \end{array}\right\} \Rightarrow I\,(x,y) = \sqrt{(\alpha^2-1)\,T^2\,(x,N\,(y))+1} + \alpha \cdot T\,(x,N\,(y))$$
2. $$\left.\begin{array}{c} I\,(x,y) = \sqrt{(\alpha^2-1)\,T^2\,(x,N\,(y))+1} + \alpha \cdot T\,(x,N\,(y)) \\ T_M\,(x,y) = \min\{x,y\} \end{array}\right\} \Rightarrow$$
3. $$I\,(x,y) = \sqrt{(\alpha^2-1)\,.\left(\min\left(x, \sqrt{(\alpha^2-1)\,y^2+1} + \alpha \cdot y\right)\right)^2 + 1} + \alpha \cdot \min\left(x, \sqrt{(\alpha^2-1)\,y^2+1} + \alpha \cdot y\right), y \in [0,1], \alpha \leq 0$$

Procedure of the first objective

Examples (The implication changed when parameter α changed)

If parameter $\alpha = 0$ in equation a, then:

$$I(x,y) = \sqrt{1 - \left(\min\left(x, \sqrt{1-y^2}\right)\right)^2}, \quad y \in [0,1], \alpha \leq 0 \quad \text{(equation } a_0\text{)}$$

Procedure of the first objective

Examples (The implication changed when parameter α changed)

If parameter $\alpha = -3$ in equation a, then:

$$I(x,y) = \sqrt{8.\left(\min\left(x, \sqrt{8y^2+1} - 3 \cdot y\right)\right)^2 + 1} \\ -3 \cdot \min\left(x, \sqrt{8y^2+1} - 3 \cdot y\right), y \in [0,1], \alpha \leq 0 \qquad \text{(equation } a_{-3}\text{)}$$

Procedure of the second objective

Step 1

The data of the temperature and the humidity of the last five years 2015-2019
([HELLENIC NATIONAL METEOROLOGICAL SERVICE 2019]) were classified in ascending order

Procedure of the second objective

Step 2

Fuzzy sets with trapezoidal membership functions were used in order to be normalized between the values [0, 1].
The selection of the membership functions was made empirically.
The language variables used were low, medium and high for the temperature and the humidity respectively.

Procedure of the second objective

Continuation of step 2

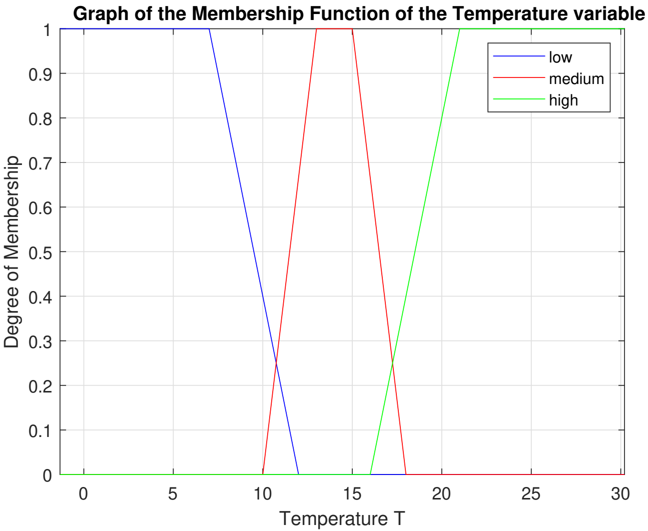
The language rules used were the following:

1. Low temperature means High humidity
2. Medium temperature means Medium humidity
3. High temperature means Low humidity.

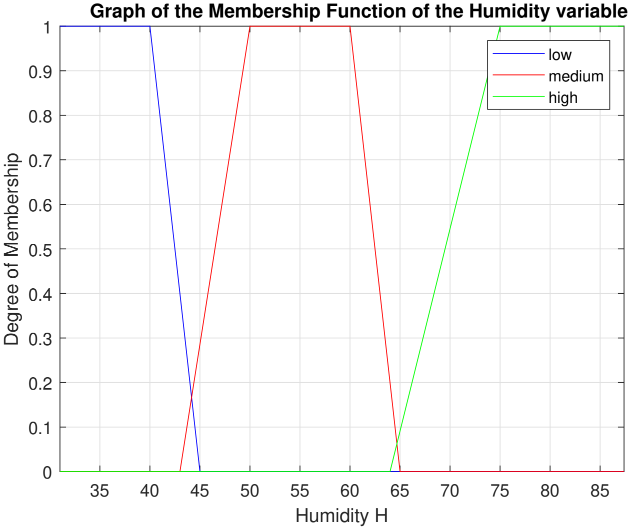
In the future:

In future work other ways will be checked in order to select the membership functions as well as the intervals where the language variables are separated.

Membership Function Temperature



Membership Function Humidity



Procedure of the second objective

Step 3

Then, the data were divided into 11 classes applying the Sturges formula (4).

$$c = 1 + \log_2 n \overset{n=780}{\Leftrightarrow} c = 1 + \log_2(780) \Leftrightarrow c = 1 + \frac{\log(780)}{\log(2)} \Leftrightarrow c = 10.6 \quad (4)$$

Procedure of the second objective

Step 4

Next, the medians of the above classes were calculated.

Procedure of the second objective

Step 5

This resulted in a 12x12 table which includes the medians of the humidity classes in the 1st row and the medians of the temperature classes in the 1st column.

Table which includes the medians

Table: Final Table

NaN	0.5857	1	1	1	0.4000	0.1564	0.3682	0.5800	0.8382	1	1
1	2	0	1	3	2	4	8	8	13	14	16
0.6020	1	0	1	5	4	6	14	5	9	15	11
0.3967	0	0	0	4	3	8	7	10	8	10	21
1	0	1	3	4	5	10	7	10	13	12	6
0.8567	1	3	6	3	9	7	6	11	9	8	8
0.4033	3	2	10	6	14	4	3	12	4	8	5
0.8220	4	4	9	9	9	12	8	6	5	3	2
1	6	9	13	14	6	7	6	3	5	1	1
1	8	16	10	8	8	9	6	2	4	0	0
1	19	12	11	11	5	3	5	4	1	0	0
1	27	24	7	4	6	1	1	0	0	0	0

Procedure of the second objective

Step 6

We assumed we wanted to fill in the cell (1 10) of the above table.

Procedure of the second objective

Step 7

The pairs of the data whose temperatures belong to the 1st class and the humidities to the 10th class were checked.

Construct Table of the Empiristic Implication

Examples

Calculation of the $Cell(1\,10)$: $\frac{N_{Cell(1,\ 10)}}{S_{Column10}} = \frac{14}{71} = 0.1972$

Table: Cell (1 10)

6.31	1	75.87	10
1.58	1	76.69	10
5.05	1	76.11	10
5.83	1	74.92	10
2.26	1	75.09	10
4.55	1	75.30	10
6.37	1	77.58	10
5.42	1	77.71	10
4.87	1	77.89	10
5.66	1	77.25	10
6.20	1	75.52	10
4.40	1	76.21	10
6.20	1	75.93	10
5.33	1	76.53	10

Procedure of the second objective

Step 8

After this check 14 pairs were found to meet the above requirement, as shown in the table above.

Procedure of the second objective

Step 9

The table was completed with a similar procedure.

Procedure of the second objective

Step 10

The resulting number for each cell was divided by the sum of the corresponding column where it belongs.

Procedure of the second objective

Step 11

From the above procedure, a new table 11x11 derived, which is the table of the empiristic implication.

Construct Table of the Empiristic Implication

Table: Table of the Empiristic Implication

0.0282	0	0.0141	0.0423	0.0282	0.0563	0.1127	0.1127	0.1831	0.1972	0.2286
0.0141	0	0.0141	0.0704	0.0563	0.0845	0.1972	0.0704	0.1268	0.2113	0.1571
0	0	0	0.0563	0.0423	0.1127	0.0986	0.1408	0.1127	0.1408	0.3000
0	0.0141	0.0423	0.0563	0.0704	0.1408	0.0986	0.1408	0.1831	0.1690	0.0857
0.0141	0.0423	0.0845	0.0423	0.1268	0.0986	0.0845	0.1549	0.1268	0.1127	0.1143
0.0423	0.0282	0.1408	0.0845	0.1972	0.0563	0.0423	0.1690	0.0563	0.1127	0.0714
0.0563	0.0563	0.1268	0.1268	0.1268	0.1690	0.1127	0.0845	0.0704	0.0423	0.0286
0.0845	0.1268	0.1831	0.1972	0.0845	0.0986	0.0845	0.0423	0.0704	0.0141	0.0143
0.1127	0.2254	0.1408	0.1127	0.1127	0.1268	0.0845	0.0282	0.0563	0	0
0.2676	0.1690	0.1549	0.1549	0.0704	0.0423	0.0704	0.0563	0.0141	0	0
0.3803	0.3380	0.0986	0.0563	0.0845	0.0141	0.0141	0	0	0	0

Construct Table of the Parametric Implication

The procedure for the construction of the Parametric Implication table

The procedure that was followed for the construction of the table is the same which was used in the first five steps for the construction of the empiristic implication table.
More specifically:

The procedure for the construction of the Parametric Implication table

Step 1

1. The data of the temperature and the humidity of the last five years 2015-2019 were classified in ascending order

The procedure for the construction of the Parametric Implication table

Step 2

Fuzzy sets with trapezoidal membership functions were used in order to be normalized between the values [0, 1].
The selection of the membership functions was made empirically.

The procedure for the construction of the Parametric Implication table

Continuation of step 2

The language variables used were low, medium and high for the temperature and the humidity respectively.
The language rules used were the following:
i. Low temperature means High humidity
ii. Medium temperature means Medium humidity
iii. High temperature means Low humidity.

The procedure for the construction of the Parametric Implication table

Step 3

Then, the data were divided into 11 classes applying the Sturges formula (4).

The procedure for the construction of the Parametric Implication table

Step 4

Next, the medians of the above classes were calculated.

The procedure for the construction of the Parametric Implication table

Step 5

This resulted in a 12x12 table which includes the medians of the humidity classes in the 1st row and the medians of the temperature classes in the 1st column.

The procedure for the construction of the Parametric Implication table

Step 6

The construction of this specific table was completed with the apply ication of equation a which is the parametric implication.

Construct Table of the Parametric Implication

Table: Table of the Parametric Implication which includes the medians

NaN	0.5857	1	1	1	0.4000	0.1564	0.3682	0.5800	0.8382	1	1
1	0.4143	0	0	0	0.6000	0.8436	0.6318	0.4200	0.1618	0	0
0.6020	0.4143	0.0053	0.0053	0.0053	0.6000	0.8436	0.6318	0.4200	0.1618	0.0053	0.0053
0.3967	0.4143	0.0106	0.0106	0.0106	0.6000	0.8436	0.6318	0.4200	0.1618	0.0106	0.0106
1	0.4143	0	0	0	0.6000	0.8436	0.6318	0.4200	0.1618	0	0
0.8567	0.4143	0.0016	0.0016	0.0016	0.6000	0.8436	0.6318	0.4200	0.1618	0.0016	0.0016
0.4033	0.4143	0.0104	0.0104	0.0104	0.6000	0.8436	0.6318	0.4200	0.1618	0.0104	0.0104
0.8220	0.4143	0.0020	0.0020	0.0020	0.6000	0.8436	0.6318	0.4200	0.1618	0.0020	0.0020
1	0.4143	0	0	0	0.6000	0.8436	0.6318	0.4200	0.1618	0	0
1	0.4143	0	0	0	0.6000	0.8436	0.6318	0.4200	0.1618	0	0
1	0.4143	0	0	0	0.6000	0.8436	0.6318	0.4200	0.16181	0	0
1	0.4143	0	0	0	0.6000	0.8436	0.6318	0.4200	0.1618	0	0

Procedure of the third objective

The programming part

The programming part of the work was structured in four parts:

Procedure of the third objective

Part 1

The first part concerned data processing, that is:

1. finding their maximum and minimum values,
2. classifying them in ascending order,
3. dividing them into classes,
4. finding the medians of the classes and
5. normalizing them by applying the trapezoidal membership functions.

Procedure of the third objective

Part 2

The second part included the construction of the empiristic table that represents the empiristic implication.

Procedure of the third objective

Part 3

The third part included the construction of the table that represents the parametric implication.

Matlab Code

```
% The OtherTable is the table of the parametric implication
OtherTable=Zeros;
k=1;
l=0;
j=1;
for i=1:1:RowSizeRawData
    if SortedXWithClasses(i,3)==j
       l=l+1;
    else
        OtherTable(j+1,1)=median(SortedXWithClasses(k:l,1));
        k=l+1;
        l=l+1;
        j=j+1;
    end
end
```

Matlab Code

```
% The OtherTable is the table of the parametric implication
OtherTable(j+1,1)=median(SortedXWithClasses(k:l,1));
k=1;
l=0;
j=1;
for i=1:1:RowSizeRawData
    if SortedYWithClasses(i,3)==j
        l=l+1;
    else
        OtherTable(1,j+1)=median(SortedYWithClasses(k:l,1));
        k=l+1;
        l=l+1;
        j=j+1;
    end
end
OtherTable(1,j+1)=median(SortedYWithClasses(k:l,1));
OtherTable(1,1)=NaN;
```

Procedure of the third objective

Part 4

The fourth part of the work presented the comparison of the empiristic implication with the well-known implications from the literature and with the parametric implication. The comparison was made between the above two tables by finding the square error.

Matlab Code

```
% We define the parametric implication as a
Nm=10;
a=0;
b=1;
NORM=zeros(1);
RA=NORM;
while Nm>0.2 && a>-100
    a=a-0.01;
    for i=2:1:ClassesX+1
     for j=2:1:ClassesY+1
        x=OtherTable(i,1);
        y=OtherTable(1,j);
        OtherTable(i,j)=sqrt((a^2-1)*(min(x,sqrt((a^2-1)*(1-
   y)^2+1)+a*(1-y)))^2+1)+a*min(x,sqrt((a^2-1)*(1-y)^2+1)+a
   *(1-y));
     end
     end
```

Matlab Code

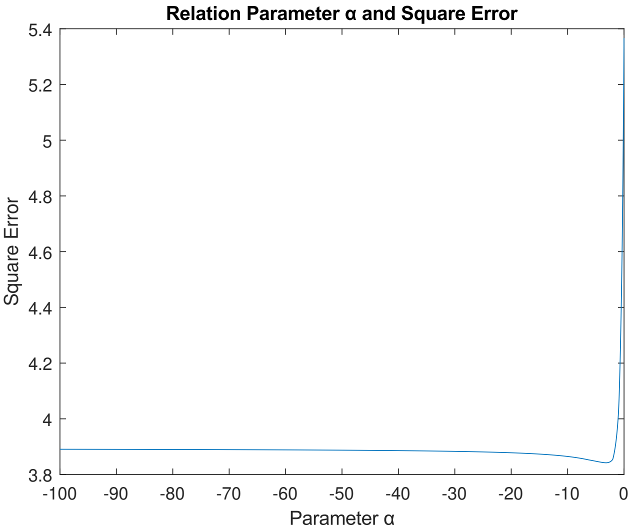
```
% We create the table of the empiristic implication
imp1=FinalTable(2:12,2:12)./sum(FinalTable(2:12,2:12));
% The control of the norm of the two implications (
    empiristic and parametric)
% We find for what value of the parameter a we have the
    minimum norm.
Nm=norm(imp1-OtherTable(2:12,2:12));
NORM(b)=Nm;
RA(b)=a;
     NORMa=vertcat(NORM,RA);
     b=b+1;
end
min(NORM);
f=find(NORM==min(NORM));
disp(NORMa(:,f))
     3.8422
    -3.1400
```

Conclusion

Part 1

The value of parameter α was also calculated, so that the parametric implication was closer to the empiristic one.

Relation between Parameter α and Square Error



Conclusion

Part 2

The implication of the smallest square error was the most appropriate.

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Thank you for your attention