Parametric Fuzzy Implications Produced Via Classes of Strong Negations

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Abstract The fuzzy implication theory has been implemented in many problems and fields. In particular, the N-negations, T-norms and I-implications concepts played crucial roles in forming the theory and applications of the fuzzy sets. The purpose of this paper is the creation of new parametric fuzzy implications via the two main fuzzy connectives, N-negations and T-norms. The N-negations used are the N^{λ}, N^{ω} and N^{α} and the conjunctions are the T_M, T_P and T_{LK}. The produced parametric fuzzy implications as well as the strategy used to create them offer more flexibility and speed in comparison to other methods of generating fuzzy implications and their products.

INTRODUCTION

The main published research based on the subject of fuzzy implications is the following: the book [1] and the papers [2], [3], [4], [5], [6] provided the definitions, properties, theorems and the classes of fuzzy implications. Furthermore, the papers [7], [8], [9] defined the fuzzy implications via automorphism functions. Finally, the book [10] and the papers [11], [12], [13], [14], [15] have demonstrated a variety of fuzzy implication applications.

MATH AND EQUATIONS

The equation: $I(x,y) = N(T(x, N(y)))$, (see Corollary 2.5.31, p. 87, [1]) is a composition of the two most well known connectives, the N-negations and the T-norms. If in the N-negations's place strong negation classes (N^{λ}, N^{ω} and N^{α}) are used and in the T-norms's place fuzzy conjunctions (T_M, T_P and T_{LK}) are used, then parametric fuzzy implication generators are produced. Formula [1] can be used to generate the new I-implications.

Theorem .1. *Assume the following:*

1. *A* $N : [0,1] \to [0,1]$ *strong negation function, which can be replaced with the known from the literature fuzzy negations*
   - $N^{\lambda}(x) = \frac{1-x}{1+\lambda x}, \quad \lambda > -1$
   - $N^{\omega}(x) = \sqrt[\omega]{1-x^{\omega}}, \ \omega > 0$
   - $N^{\alpha}(x) = \sqrt{(a^2-1)x^2+1} + \alpha \cdot x, \ a \leq 0$
2. *A continuous Archemedean and strict t-norm* $T : [0,1]^2 \to [0,1]$*, which can be replaced with the known from the literature fuzzy conjunctions*

- $T_M(x,y) = \min\{x,y\}$
- $T_P(x,y) = x \cdot y$
- $T_{LK}(x,y) = \max\{x+y-1,0\}$

Then, there is a function $I_\lambda : [0,1]^2 \to [0,1]$ *which is a I-implication, such that:*

$$I_\lambda(x,y) = N^\lambda(T(x,N^\lambda(y))) \tag{1}$$

Then, there is a function $I_\omega : [0,1]^2 \to [0,1]$ *which is a I-implication, such that:*

$$I_\omega(x,y) = N^\omega(T(x,N^\omega(y))) \tag{2}$$

Then, there is a function $I_\alpha : [0,1]^2 \to [0,1]$ *which is a I-implication, such that:*

$$I_\alpha(x,y) = N^\alpha(T(x,N^\alpha(y))) \tag{3}$$

Proof. The fact that function I_λ satisfies the properties of a fuzzy implication will be proved.
Indeed:

- The function I_λ is decreasing with respect to its first variable.
  Assume the following:
  $\forall x_1,x_2,y \in [0,1]$, with $x_1 \leq x_2$ it will be shown that: $I_\lambda(x_1,y) \geq I_\lambda(x_2,y)$
  $I_\lambda(x_1,y) \geq I_\lambda(x_2,y) \Leftrightarrow N^\lambda(T(x_1,N^\lambda(y))) \geq N^\lambda(T(x_2,N^\lambda(y))) \Leftrightarrow$
  $T(x_1,N^\lambda(y)) \leq T(x_2,N^\lambda(y)) \Leftrightarrow x_1 \leq x_2$

- The function I_λ is increasing with respect to its second variable.
  Assume the following:
  $\forall y_1,y_2,x \in [0,1]$, with $y_1 \leq y_2$, it will be shown that: $I_\lambda(x,y_1) \leq I_\lambda(x,y_2)$
  $I_\lambda(x,y_1) \leq I_\lambda(x,y_2) \Leftrightarrow N^\lambda(T(x,N^\lambda(y_1))) \leq N^\lambda(T(x,N^\lambda(y_2))) \Leftrightarrow$
  $T(x,N^\lambda(y_1)) \geq T(x,N^\lambda(y_2)) \Leftrightarrow N^\lambda(y_1) \geq N^\lambda(y_2) \Leftrightarrow y_1 \leq y_2$

- The function I_λ satisfies the boundary condition: $I_\lambda(0,0) = 1$
  $I_\lambda(0,0) = N^\lambda(T(0,N^\lambda(0))) = N^\lambda(T(0,1)) = N^\lambda(0) = 1$

- The function I_λ satisfies the boundary condition: $I_\lambda(1,1) = 1$
  $I_\lambda(1,1) = N^\lambda(T(1,N^\lambda(1))) = N^\lambda(T(1,0)) = N^\lambda(0) = 1$

- The function I_λ satisfies the boundary condition: $I_\lambda(1,0) = 0$
  $I_\lambda(1,0) = N^\lambda(T(1,N^\lambda(0))) = N^\lambda(T(1,1)) = N^\lambda(1) = 0$

So, the function I_λ is a fuzzy implication. Using the same method the equations [2] and [3] are proved. □

The graph [Figure 1] shows the fuzzy implication I_λ^1 constructed via the equation [1] using N^λ and T_M.

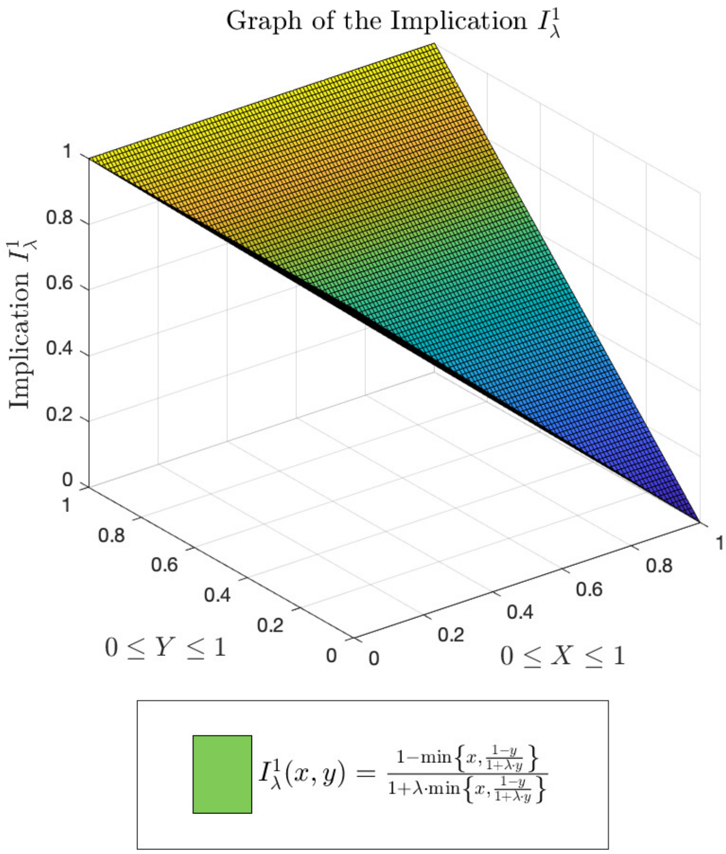

Figure 1. Graph of the Implication I_λ^1

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